

A Proposal to Define Social Distance in the context of the Covid19 Pandemic

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Abstract: This paper attempts to propose an indicator for quantifying the social distance, which unfortunately is a very popular term these days. The introduction presents 8 reasons why this indicator is fit for public extensive use, among which relevance for decision, simplicity, intuitiveness and low error. The paper **establishes the existence of a theoretical threshold value for the Social Distance** which once **surpassed determines the decrease of the Number of Active (Contagious) Cases**. The study of the situation in a number of countries **confirms the existence of the threshold**, with **values within or close to the range theoretically predicted**. Although the concept may be refined, the above reasons and the present worldwide situation, including the need to monitor a possible second wave of pandemic, justify a quick adoption in the author's opinion, considering however the need for further improvements.

Introduction: What would such an indicator useful for?

1. it is associated to a term (social distance) that has become very popular and extensively used around the world and provides a quantification for this term;
2. its significance is intuitive;
3. it may be used in measuring the progress in achieving social distance during an epidemic at a country, region and local community level;
4. it may be compared to a threshold value for social distance that insures that the number of active cases (total cases – total recovered – total deaths) starts to decrease; the difference between the threshold value and the actual social distance shows if and how intense must be the next measures to be applied in order to surpass the threshold;
5. having a social distance that is comfortably higher than the threshold value provides assurance regarding the continuous decrease of the number of active cases even if mistakes are made;
6. it is very simple to compute by a simple division of two values, obtained from already available, directly measured data;
7. it has a low sensitivity (low error) with respect to the difference in between the real number of cases and confirmed (through testing) number of cases;
8. it may be easily used in public communication in order to clearly explain the progress in combating the epidemic.

Definition

Social distance = the average time it takes a contagious person to infect another person.

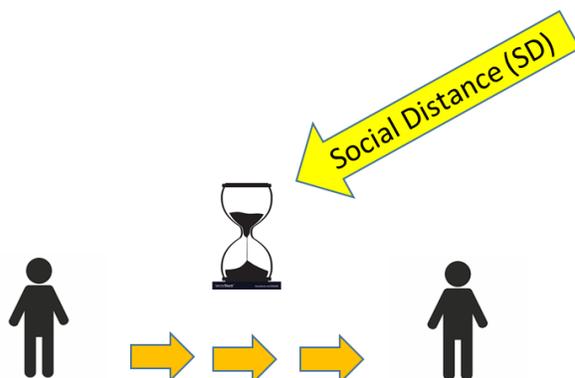


Fig. 1 – a visual metaphor for social distance

It's an intuitive definition: the later is infected (in average) another person by a contagious person, the higher we may say is the social distance achieved.

If infecting another person takes (in average) longer (the social distance is higher) than the **duration of the state of contagion – Contagiousness Duration (CD)** then it is likely that such infection would not happen.

As a side comment, the proposed Social Distance is actually a Quasi Distance (Quasi Metricⁱ) as defined in mathematics. If the time it would take A to infect B would be equal with the time it would take B to infect A (symmetry) then we would have an actual distance in mathematical terms.

The formula

Given the above definition, the formula for Social Distance is as follows:

$$\mathbf{SD} = \frac{\Delta t}{\widetilde{\Delta N_c(t)}} N_a(t-t_i) = \frac{N_a(t-t_i)}{\widetilde{\Delta N_c(t)}} \quad (1)$$

Where:

Δt - is the time interval at which the variation of new cases is considered (chosen as follows to be $\Delta t = 1$ day)

$\Delta N_c(t)$ – is the number of new cases (which are at the same time contagious) registered in day “t” compared to the previous day “t-1”.

$\widetilde{\Delta N_c(t)}$ – is the moving averageⁱⁱ of $\Delta N_c(t)$. This has been considered in order to smoothen the $\Delta N_c(t)$ function knowing that the moving average acts like a low-pass filter.

$N_a(t-t_i)$ – is the number of active (contagious) cases previously registered in day “t-t_i” where t is today and t_i is the incubation period.

This model is a simplifying one in order to obtain a simple formula. In fact, the number of new cases confirmed through testing in a certain day is determined by the number of contagious cases from several previous consecutive days. We used the average incubation period as if all the causes of the new cases in day “t” would have been concentrated in the same “t-t_i” day.

Explaining the formula

Fig. 2 (below) – The number of active cases (which are as well contagious) at “t-t_i”, noted with “ $N_a(t-t_i)$ ”

- determines by infecting other persons,
- after the average incubation period “t_i”, which is at moment “t”,
- during one day,
- a number of new cases equal to “ $\Delta N_c(t)$ ”.

As this last function is jagged (has high frequency components), in order to extract the trend and eliminate noise, the function has been low-pass filtered by using a 5 days moving average function – noted $\widetilde{\Delta N_c(t)}$.

Explaining the Social Distance formula.

If the $N_a(t-t_i)$ persons that are contagious at moment “t-t_i”,

- cause during one day (day “t”) the appearance of “ $\widetilde{\Delta N_c(t)}$ ” new cases,
- one single person of the $N_a(t-t_i)$ contagious persons will determine during day „t” $N_a(t-t_i)$ times less cases, i.e. $\frac{\widetilde{\Delta N_c(t)}}{N_a(t-t_i)}$ new cases.

If a person determines $\frac{\widetilde{\Delta N_c(t)}}{N_a(t-t_i)}$ cases during day „t”, a new case would be caused by one person (which is the above definition of Social Distance) every

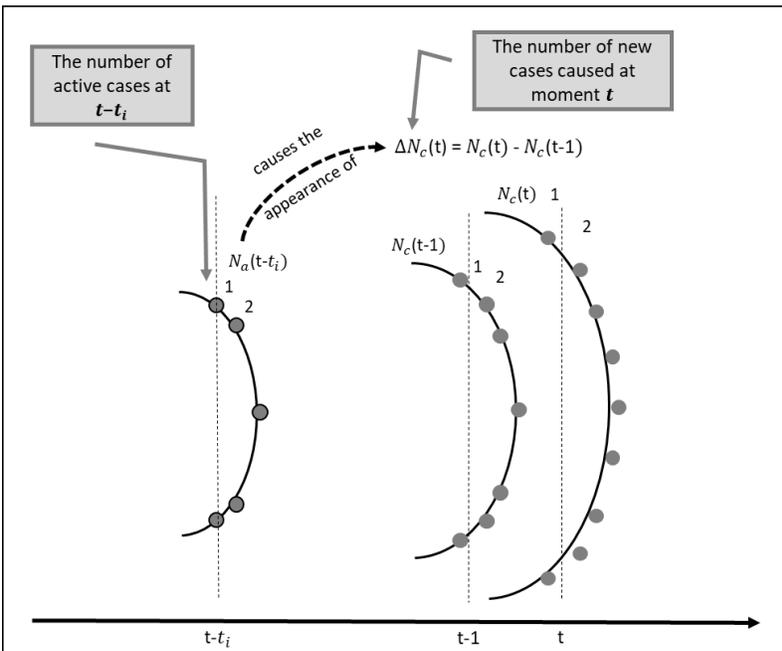


Fig 2. Explaining the Social Distance formula.

$$SD = \frac{1 \text{ day} * 1 \text{ case}}{\frac{\Delta \bar{N}_c(t)}{N_a(t-t_i)}} = \frac{N_a(t-t_i)}{\Delta \bar{N}_c(t)} \text{ days.} \quad (2)$$

The number of active cases at $t-t_i$

The filtered number of new cases at moment "t"

Explaining The Social Distance Threshold Associated to the Decrease of Active Cases

If the Social Distance (the period of time it takes a contagious person to infect another person) is less than the average period of contagion – $SD < CD$ – during the period of contagion, the contagious person will infect in average more than one person and the result will be the growth of the number of active (contagious) cases.

For example, if

- $SD=10$ (the average period it takes to infect another person) and
- $CD = 30$ (period of contagion),
- during the 30 days period of contagion,
- $30/10 = 3$ other persons shall be infected.

If $SD = CD$, a contagious person shall infect in average just one other person during the period of contagion, maintaining this way the number of contagious person at the same value.

Finally, if $SD > CD$, a contagious person shall infect in average less than one person and this shall determine the decrease of the number of active / contagious cases in time.

The Source for the Variability of the Contagiousness Duration = Social Distance Threshold Value

It is expected to see that different countries show different Social Distance Threshold values (Contagiousness Duration). The following analyzes the possible sources of such variability.

Each new infected case goes through different stages during which the time it takes to infect another person (the social distance) is different. Let's examine the process.

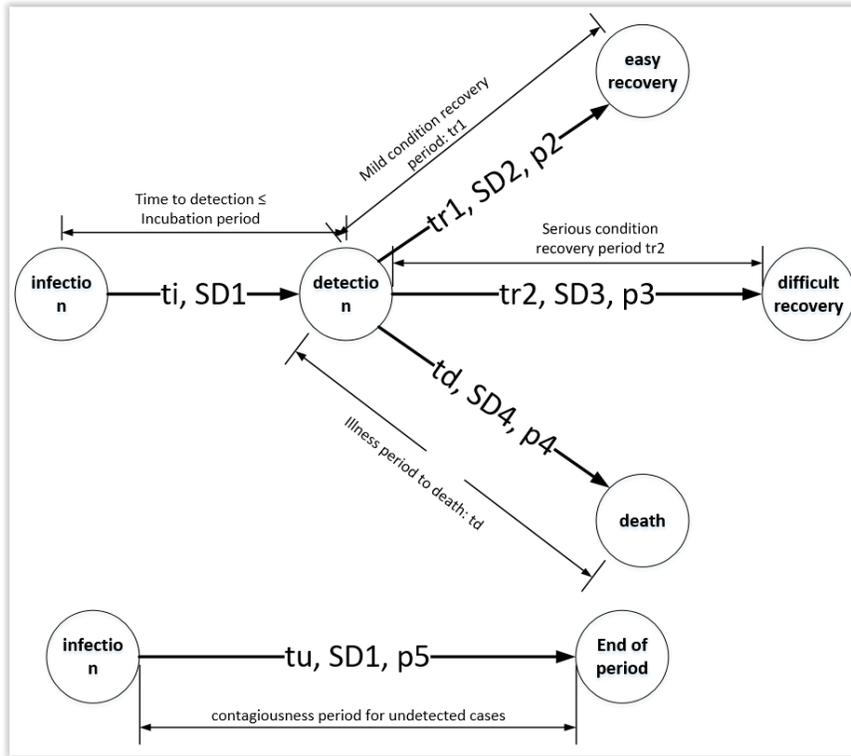


Fig. 3 Contagiousness process:

Cases may be detected or undetected. Detected cases go through the incubation period (stage 1) and may follow three evolution paths:

- easy recovery,
- difficult recovery and
- death.

Each path is characterized by a specific duration, Social Distance and probability.

Undetected cases go through a single stage (asymptomatic), characterized by a duration and a Social Distance that is probably similar to the one specific to the incubation period for the detected cases (similar social conditions).

The overall Social Distance (SD) for the detected cases is the total time $\frac{t_t}{N_i}$ (duration) of the process divided by the number of persons infected N_i (time needed to infect 1 person):

$$SD = \frac{t_t}{N_i} = \frac{\tilde{t}_t}{1} \quad (3)$$

The average time \tilde{t}_t to infect one person is given by:

$$\tilde{t}_t = \tilde{t}_i + p_2 * \tilde{t}_{r1} + p_3 * \tilde{t}_{r2} + p_4 * \tilde{t}_d \quad (4)$$

where $p_2 + p_3 + p_4 + p_5 = 100\%$

Table 1, below shows the values considered in the Monte-Carlo evaluation of the CD values distribution.					
No	COVID19 evolution ^{iii, iv}	% of cases with such evolution	Time to detection (days)	Time to recovery / to death	Remarks
1	Asymptomatic	37.5% ± 11.25%	For asymptomatic cases, the CD was considered as an independent random value with the same variation interval as for the Mild cases		
2	Mild	42.5% ± 11.25%	2-14 days	7-14 days	1+2 = 80%
3	Severe & critical that survive	16% ± 1.2%		21-42 days	3+4 = 20%
4	Deaths	4% ± 1.2%		14-56 days	

Based on the above data, we generated 10000 cases of Contagiousness Duration value. Their distribution is shown below in Figure 4. The following relevant values were obtained (10000 cases):

Minimum Contagiousness Duration: 12.72 days (minimum minimorum – 2+7 = 9 days)

Maximum Contagiousness Duration: 38.13 days (maximum maximorum – 14+56 = 70 days)

Average Contagiousness Duration: 25.60 days

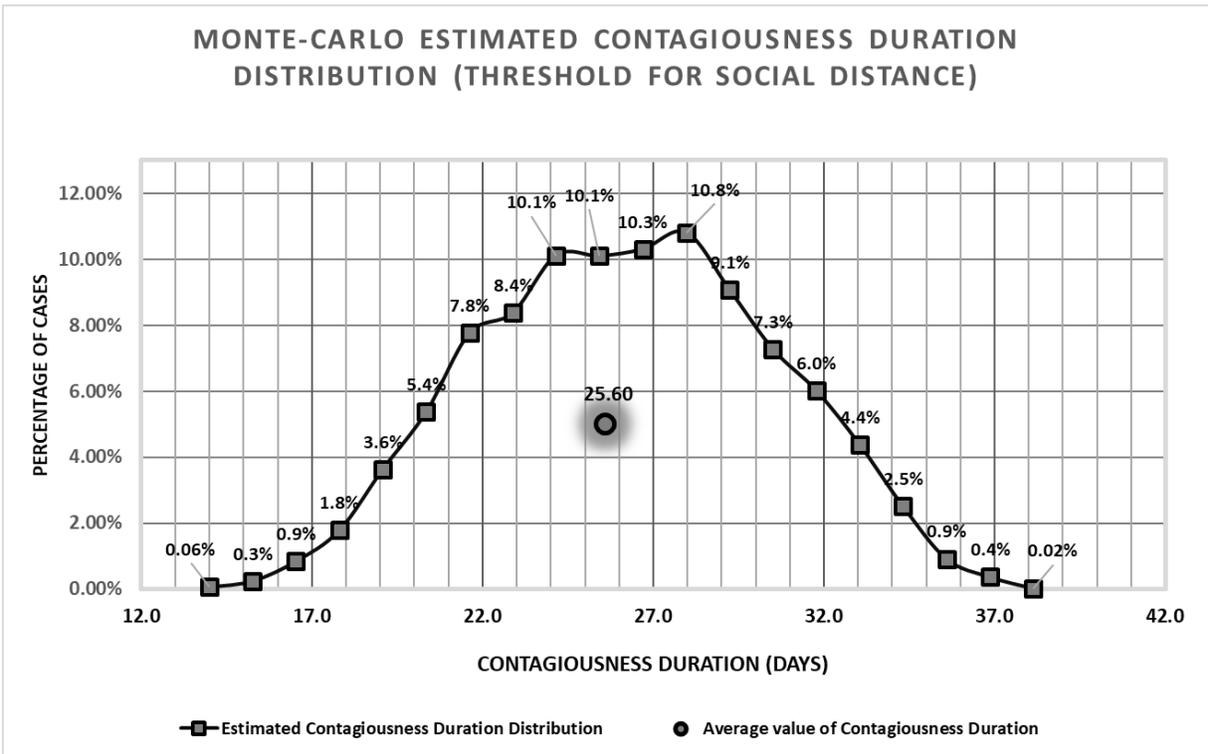


Fig. 4 – the Monte-Carlo generated distribution for the Contagiousness Duration = Threshold for the Social Distance that determines the decrease of the number of active cases. Number of cases randomly generated: 10000

In fact, there are factors that influence the above taken into consideration random variables (they are not authentic random variables). Some of them may be directly determined by the measures taken and the capability of the medical system to deal with the pandemic (marked in light gray color). The next table depicts some of these factors.

Parameter	Influencing factor(s)
Time to detection	Rigorous case/contacts tracing Number of tests performed
Recovery duration	Age Individual variability, including pre-existing co-morbidities Medical protocol / system capability
Illness period to death	Age Individual variability, including pre-existing co-morbidities Medical protocol / system capability
Contagiousness period for undetected cases	Age Individual variability, including pre-existing co-morbidities
Probability to have undetected cases	Number of tests performed
Probability of easy recovery	Age and pathological profile of the infected population Medical protocol / system capability
Probability of serious or critical condition	Age and pathological profile of the infected population Medical protocol / system capability
Probability of death	Age and pathological profile of the infected population Medical protocol / system capability

Based on the above, we would expect that countries with strong governance and medical systems would have lower CD values.

The same distribution shows us that if a country would consider a **level of confidence of 80%** regarding the **decrease of the number of active cases**, it should consider **the value of 30 days for the Social Distancing threshold**.

If a country would consider a **level of confidence of 95%** regarding the **decrease of the number of active cases**, it should consider **the value of 35 days for the Social Distancing threshold**.

The above thresholds do not take in consideration the error in measuring the Social Distance based only on the confirmed (through testing) cases. Chapter “Estimating the Error in Measuring the Social Distance through the Proposed Formula” estimates the value of this error.

The Relationship between Social Distance and the Number of Active Cases in a Number of Countries shows the Existence of a Threshold Value Close to the Theoretically Predicted One

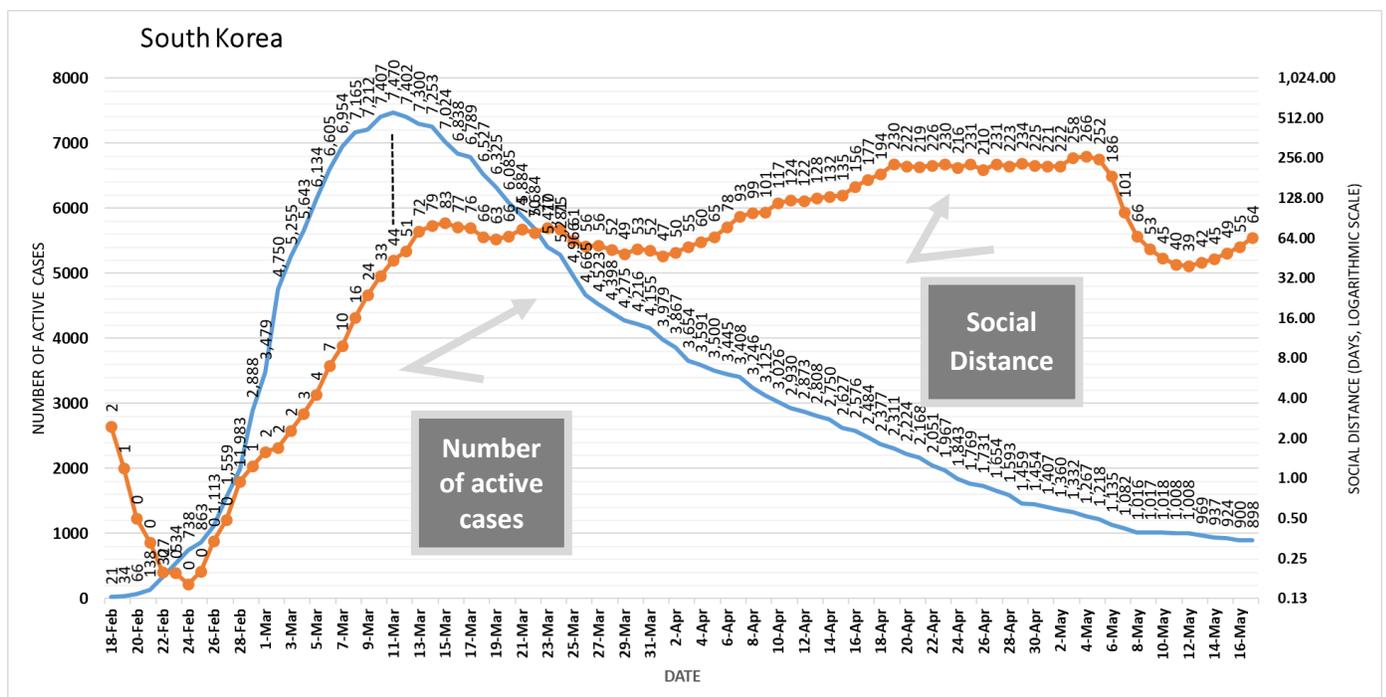
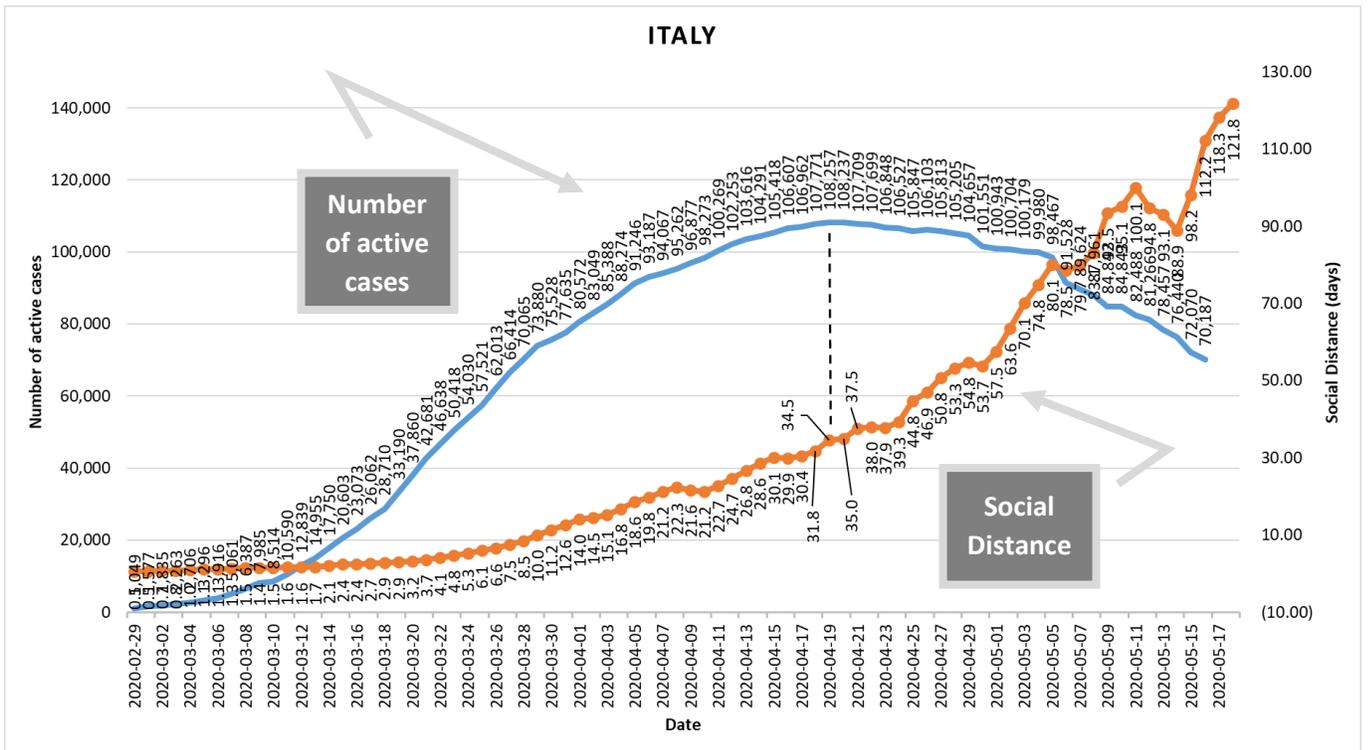


Fig 5. – South Korea was the first country to register a decrease of the number of active cases at the moment when the Social Distance leaped in one day from 33 to 44 (days) and then to 51 and 72. Data source: ^{vi}



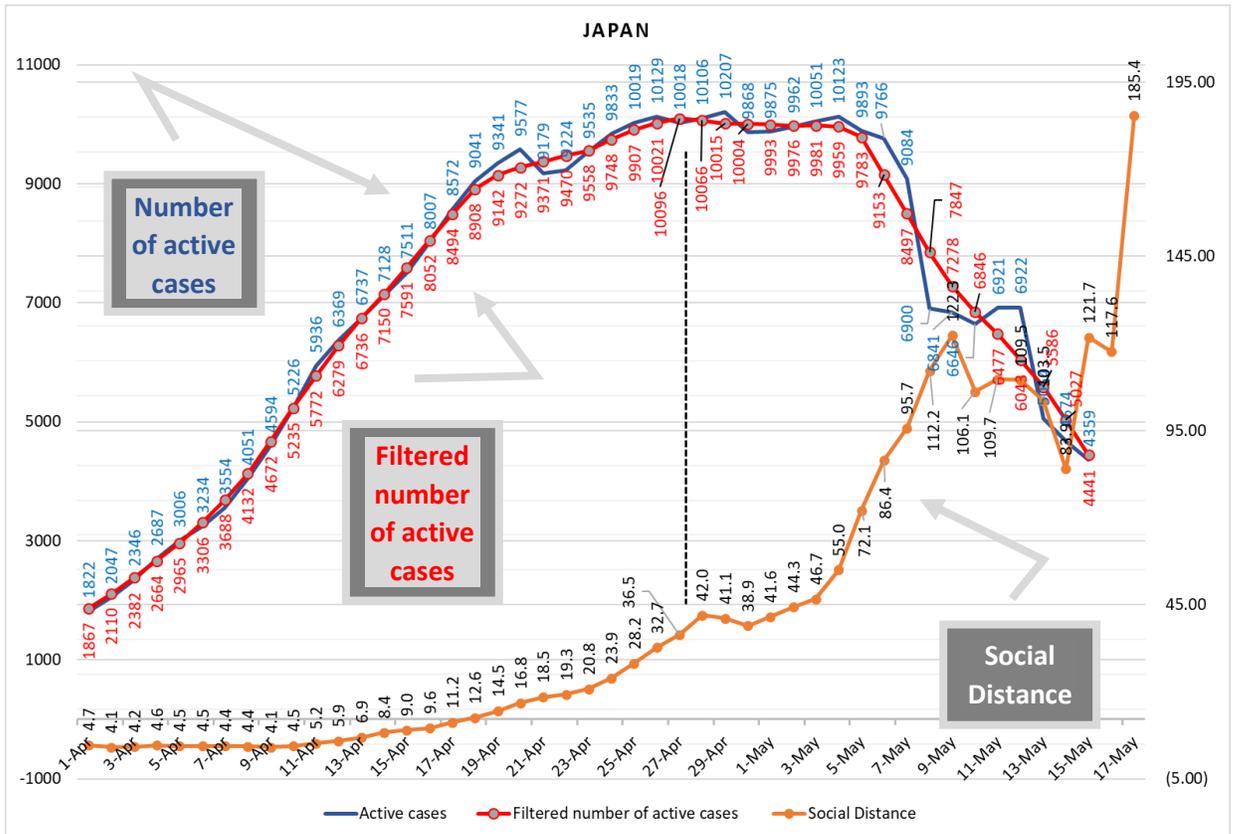


Fig. 8 – Japan had a jagged plateau (in terms of active cases) which also corresponded with a “hesitant” growth of the Social Distance. A maximum of the number of active cases, corresponding to 36.5 days was identified. Data source:^{ix} To be able to identify this maximum we used a 5 days moving average low-pass filter that was applied to the active cases curve.

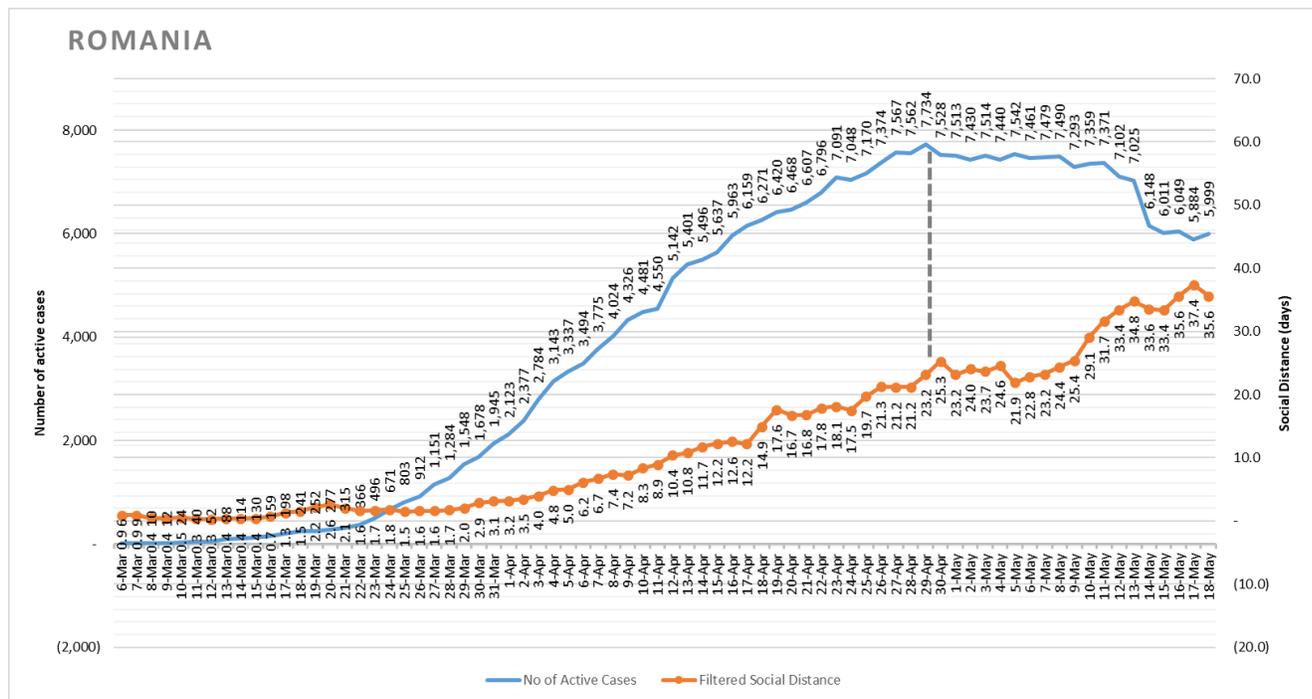


Fig. 9 – Romania also showed a jagged plateau of the number of Active Cases. A timid decrease started at SD=25.3. Only after reaching SD=34.8, the decrease had a higher slope. Data source: ^x

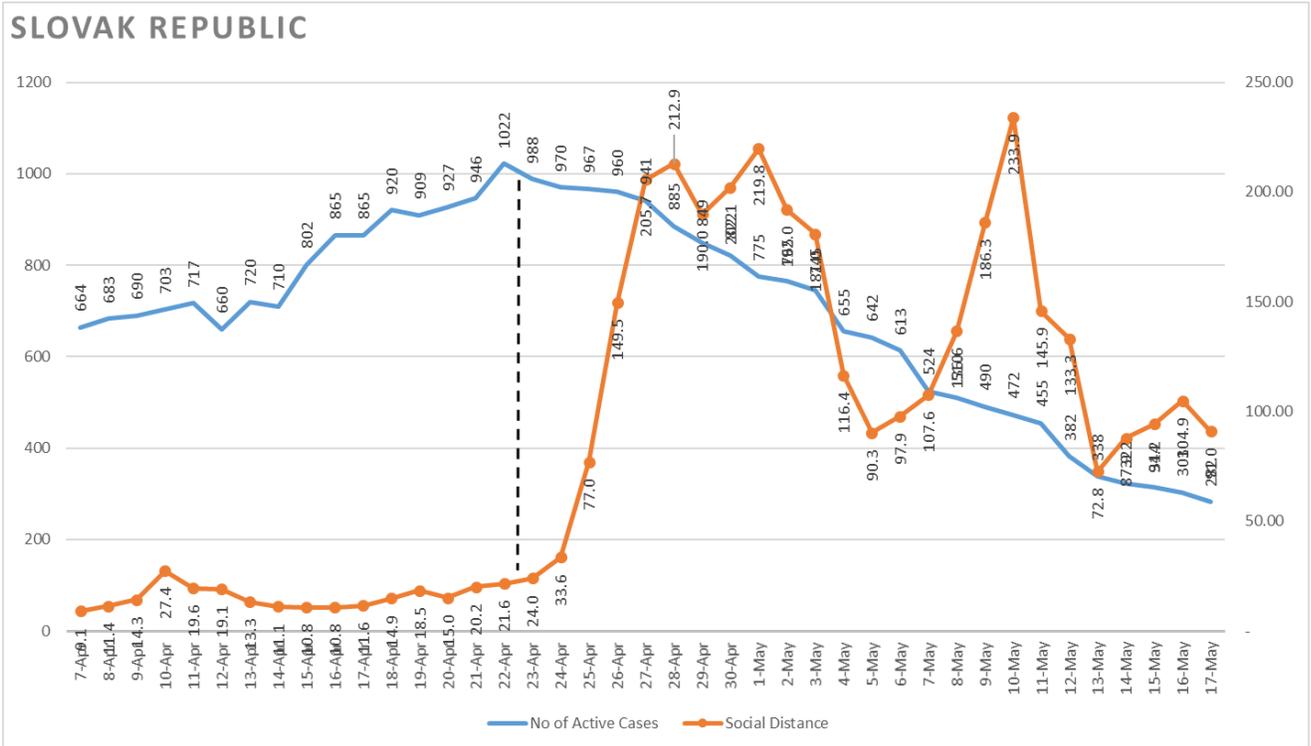


Fig. 10 The decrease of the Number of Active Cases in the Slovak Republic starts as soon as the Social Distance reaches 21.6 days (followed by a steep ramp of the Social Distance up to over 200 days). Data source: ^{xi}

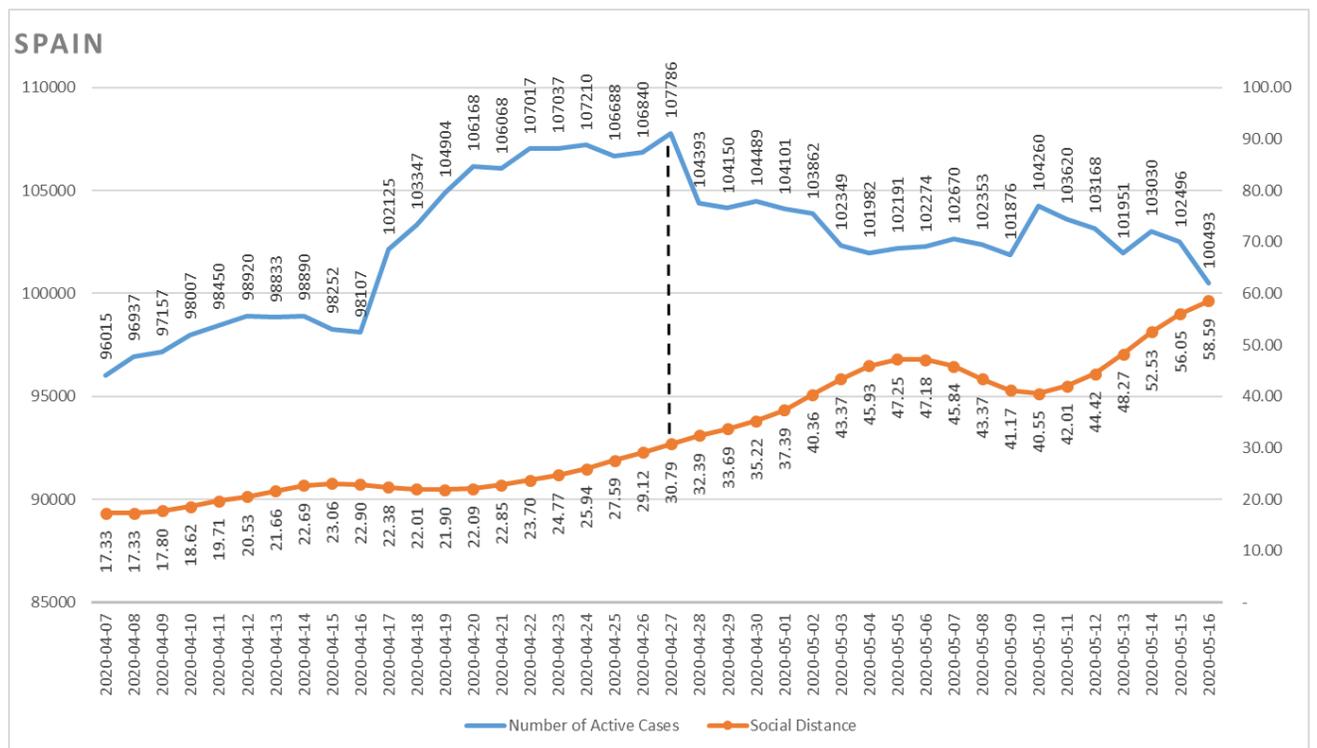


Fig. 11 – Spain also showed a decline of its number of active cases once it reached SD=30.79. The reduction of SD around may 10th corresponds to a new local maximum of active cases. Data Source ^{xii}

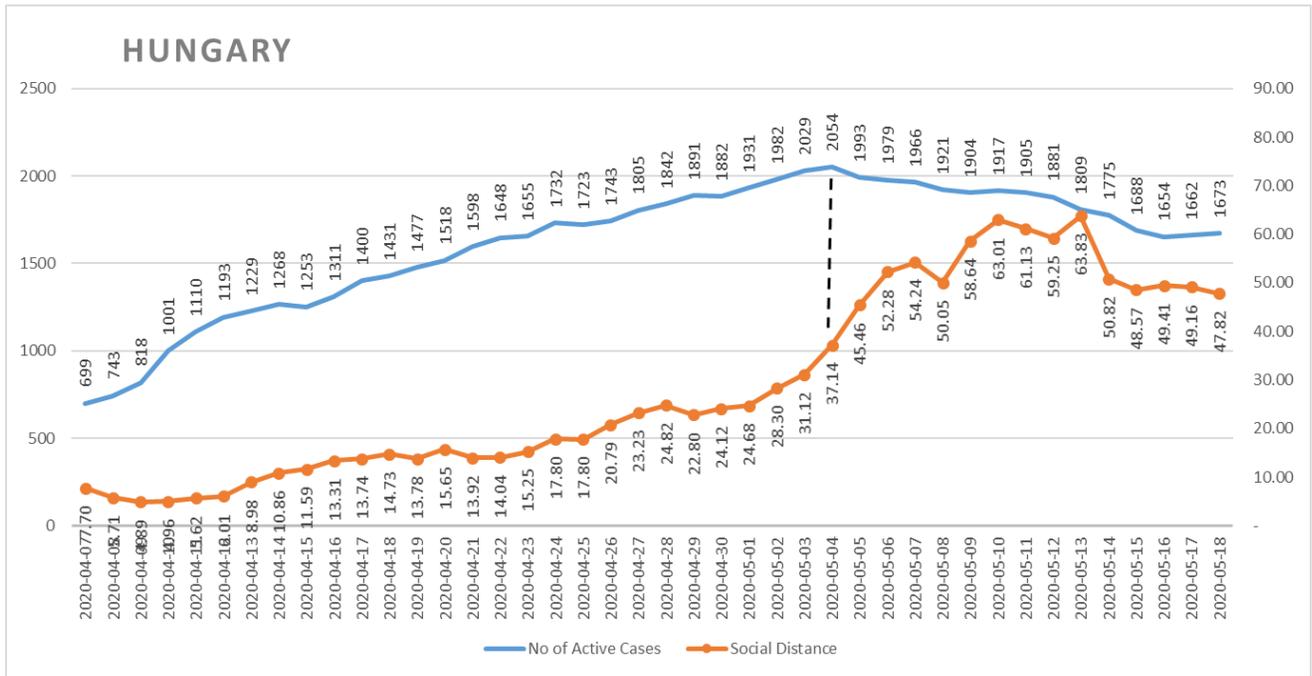


Fig. 12 – Hungary’s number of Active Cases started to decline once SD surpassed 37.14 days. Data Source: ^{xiii}

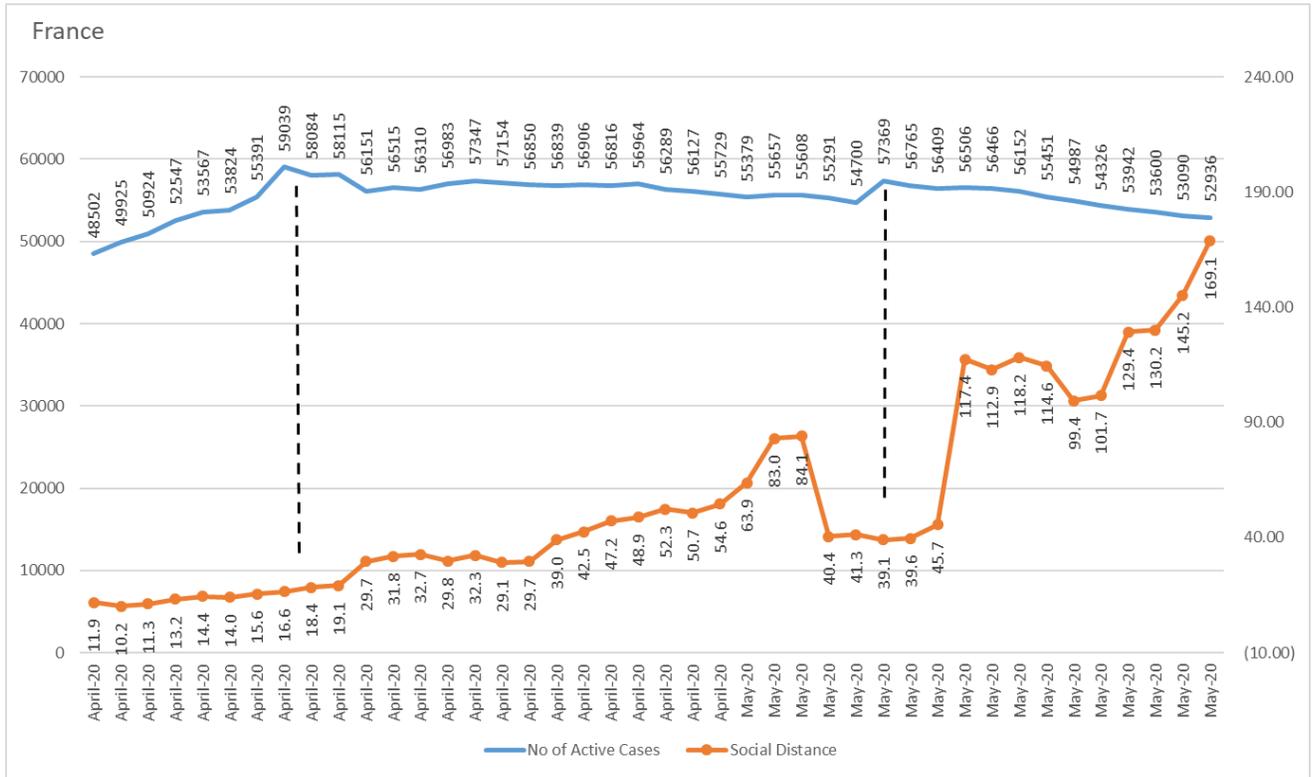


Fig. 13 – France has a two-staged decrease of the number of active cases: initially, it starts a timid decrease when SD=16.6, followed by a second stage of decrease at SD=39.1, after what seems to be a correction of data regarding the number of cases by over 2500 cases. Data source ^{xiv}

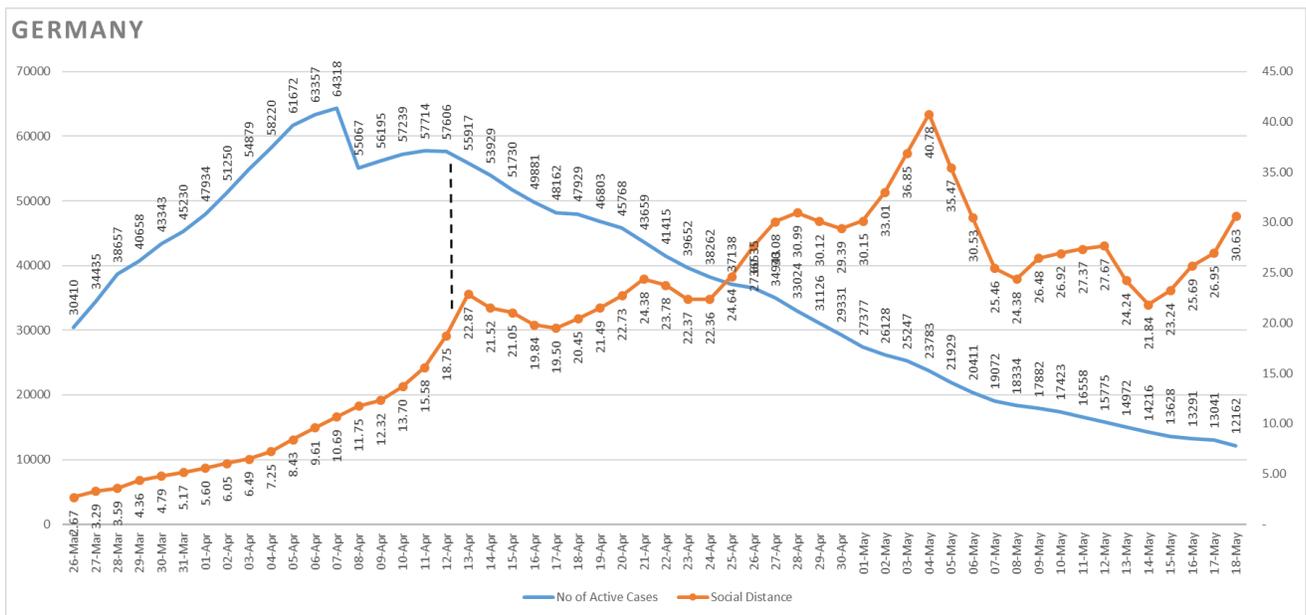


Fig. 14 – Germany shows to maximum values for the number of active cases. While the first maximum is followed immediately by an 14.3% decrease (which would be rather the result of changing the way the active cases are counted), the second maximum corresponds with the leap from 18.75 to 22.87 days in Social Distance followed by a slow decrease of the number of active cases. The values are within the theoretically predicted ones even though the SD threshold is low. This may be explained by the large number of tests performed and the high quality of the health system which are both factors that reduce the threshold value. Data source: ^{xv}

Two examples are situated at the extremes regarding the Social Distance Threshold

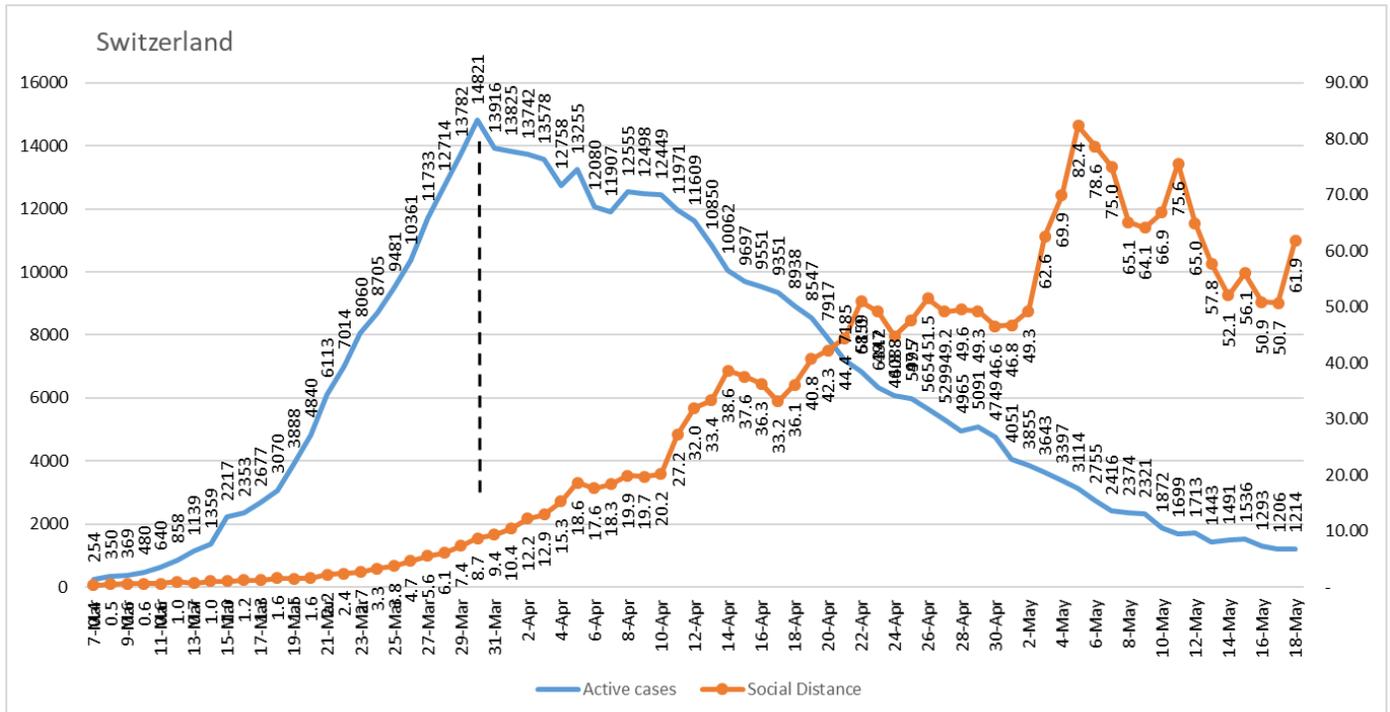


Fig. 15 – In Switzerland, the number of active cases starts to decrease as soon as Social Distance reaches 8.7 days (just below the minimum-minimorum). However, 4 days later SD becomes 12.9 which is the minimum value obtained through the Monte-Carlo simulation. We should keep in mind that the CD is not really a random variable and may be influenced by factors such as the ones identified earlier in this paper as well as by the error level estimated later in this paper. Data source: ^{xvi}

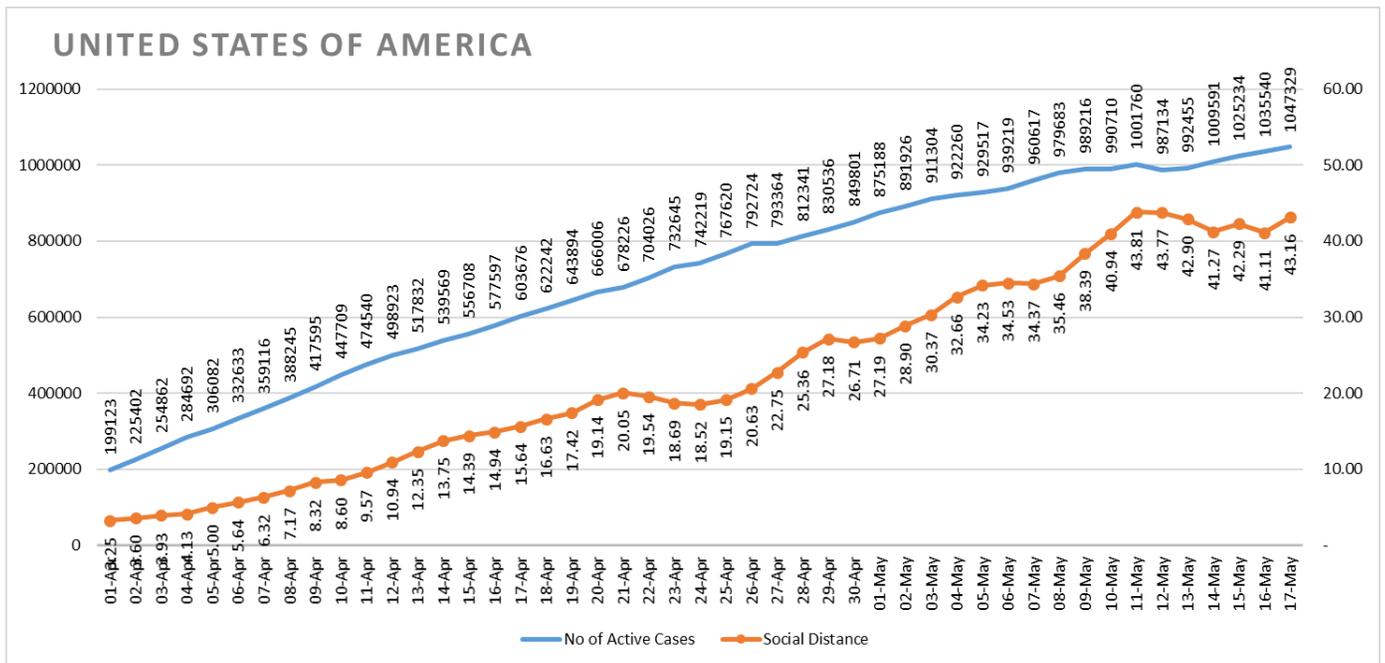


Fig. 16 – The number of Active Cases in the US seems to reach a plateau (even started to decrease when SD = 43.77 days). However, the value is over the threshold value determined through the Monte Carlo simulation but well below the maximum-maximorum value. Data source: ^{xvii}

Estimating the Error in Measuring the Social Distance through the Proposed Formula

Estimating the Social Distance error due to the difference between the real number and confirmed through testing number of cases.

The above formula for the Social Distance is based on measuring two parameters:

- the past (t_i days ago) number of active (contagious) cases and
- today's number of new cases.

Both numbers may be erroneous as they depend on the way testing is being performed.

A case is declared active (and is also contagious) as soon as a test is positive. A number of asymptomatic cases may not be tested and may be missed even if contagious. The exit from the “active” state is certain as it is based either on two consecutive negative tests or, unfortunately, through the death of the patient. Thus, the number of active cases may be erroneous due to the way we measure the number of new cases.

As already mentioned above, the number of new cases is based on the exact same principle – once a person tests positive, the person becomes a “new case” and is considered “active”. Again, a number of asymptomatic cases may not be tested and may be missed even if contagious.

So, both parameters are affected **by the same cause of error**: the way we measure the number of new cases.

The only in difference between the two errors is due to the fact that the two **measurements are made at “ t_i ” days distance** and we may assume that the error in day “ $t-t_i$ ” may be different from the error in day “ t ”.

We may conclude that the Social Distance precision is affected twice by the same cause in two different ways as two causes (of identical nature) generate different effects at a few days distance.

This consideration is important as follows in estimating the Social Distance error: two errors generated by the same type of cause at a few days distance may not be very far apart one from each other.

Let's suppose that the real value of the active cases at the moment " $t-t_i$ " is

$$[N_a(t-t_i)]_R = (1 + \varepsilon_{t-t_i}) * N_a(t-t_i) \quad (3)$$

where ε_{t-t_i} is the percentage value of the error that affects the number of active cases at moment " $t-t_i$ " and the real value of the new cases at moment " t " is

$$[\Delta N_c]_R = (1 + \varepsilon_t) * \Delta N_c \quad (4)$$

In this case:

$$[SD]_R = \frac{[N_a(t-t_i)]_R}{[\Delta N_c(t)]_R} = \frac{(1 + \varepsilon_{t-t_i}) * N_a(t-t_i)}{(1 + \varepsilon_t) * \Delta N_c} = \frac{(1 + \varepsilon_{t-t_i})}{(1 + \varepsilon_t)} * SD \quad (5)$$

In conclusion, the relative error affecting the Social Distance (ε_{SD}) is:

$$\varepsilon_{SD} = ABS\left(\frac{[SD]_R - SD}{[SD]_R}\right) = ABS\left(1 - \frac{SD}{[SD]_R}\right) = ABS\left(1 - \frac{(1 + \varepsilon_t)}{(1 + \varepsilon_{t-t_i})}\right)$$

so:

$$\varepsilon_{SD} = ABS\left(\frac{(1 + \varepsilon_{t-t_i})}{(1 + \varepsilon_t)} - 1\right) \quad (6)$$

Hypotheses regarding ε_{t-t_i} and ε_t

The above conclusion that "two errors generated by the same type of cause at a few days distance may not be very far apart one from each other" offers the following possible path in analyzing the Social Distance error based on the two variables ε_{t-t_i} and ε_t representing the errors related to the number of new cases.

Also, both errors **are to be considered positive or zero** (the number of real cases is larger than the one measured with tests) as we considered that the tests are certain when they identify an infected person (there are no or irrelevantly few false positive situations).

If $\varepsilon_{t-t_i} = \varepsilon_t$, $\varepsilon_{SD} = 0$

If $\varepsilon_{t-t_i} \neq \varepsilon_t$ let's consider that if $\varepsilon_{t-t_i} = \varepsilon_t + \delta$ where $\delta \in \mathbb{R}^*$, then

$$\varepsilon_{SD} = ABS\left(\frac{(1 + \varepsilon_t + \delta)}{(1 + \varepsilon_t)} - 1\right) = ABS\left(\frac{(\delta)}{(1 + \varepsilon_t)}\right) \quad (7)$$

The maximum of this function (knowing that $\varepsilon_t > 0$) is $ABS(\delta)$ when $\varepsilon_t = 0$ and $\varepsilon_{t-t_i} = \delta$, so:

$$MAX(\varepsilon_{SD}) = ABS(\delta) \quad (8)$$

What is remarkable is the fact that the larger today's error ε_t in measuring new cases the smaller is ε_{SD} , the difference δ between the two errors is proportional (and the superior limit) of ε_{SD} .

This means that for larger values of ε_t , keeping the same difference between errors δ , ε_{SD} becomes smaller.

For illustration purposes, the average value of ε_{SD} (noted $\overline{\varepsilon_{SD}}$) was computed for different δ and a given $\varepsilon_t = 200\%$ to show that in this case, the average error in determining the Social Distance is roughly half of the difference of the error in determining the number of new cases in two moments of time separated by the incubation period.

δ	$\overline{\varepsilon_{SD}}$ (%)	$\overline{\varepsilon_{SD}}$ (days)*	δ	$\overline{\varepsilon_{SD}}$ (%)	$\overline{\varepsilon_{SD}}$ (days)*	δ	$\overline{\varepsilon_{SD}}$ (%)	$\overline{\varepsilon_{SD}}$ (days)*
1.0%	0.5%	0.2	8.0%	4.4%	1.7	15.0%	8.1%	3.1
2.0%	1.1%	0.4	9.0%	4.9%	1.9	16.0%	8.7%	3.3
3.0%	1.6%	0.6	10.0%	5.4%	2.1	17.0%	9.2%	3.5
4.0%	2.2%	0.8	11.0%	6.0%	2.3	18.0%	9.8%	3.7
5.0%	2.7%	1.0	12.0%	6.5%	2.5	19.0%	10.3%	3.9
6.0%	3.3%	1.3	13.0%	7.1%	2.7	20.0%	10.8%	4.1
7.0%	3.8%	1.5	14.0%	7.6%	2.9			

It is reasonable to believe (with the exception of change of testing policy) that the difference δ between the two errors in determining the number new cases would be small (even if the errors themselves would be large).

If for example for an incubation period of 6 days $\varepsilon_{t-t_i}=490\%$ and $\varepsilon_t=485\%$, $\overline{\varepsilon_{SD}} = 0.9\%$.

Enlarging the example, if the **difference in errors** δ would be 5% and $\varepsilon_t = 0\%$, ... **1000%**, the average value of the error in determining the Social Distance would be $\overline{\varepsilon_{SD}} = 1.2\%$, a value more than reasonable.

The same average value of the error in determining the Social Distance for a difference of 10% between the errors on new cases, while $\varepsilon_t = 0\%$, ... **400%**, would be 3.99%.

Conclusions and further directions of development

The proposed indicator is fit for use and making good decisions and as well as for effective public communication.

At the time this paper was developed the information was available only related to a small number of examples of countries with decreasing number of active cases in order to confirm the theoretical results of this paper. Further confirmation is necessary. Also data used had as main source Wikipedia – the level of confidence in this data may be subject of discussion.

The indicator may be refined (e.g. the number of new cases in one day is caused by the number of active cases in several previous consecutive days), this model uses the average incubation period as if all the causes of new cases in day "t" were concentrated in the same " $t-t_i$ " day, where t_i is the incubation period.

Further investigation is needed to explain the difference for the two extreme cases situations (Switzerland and the US) between the theoretical value of the threshold (given by the average

contagiousness period) and the one actually determined from the evolution curves of the Filtered Number of Active Cases vs the computed Social Distance.

For the moment being, using a low-pass filter for the Number of New Cases seemed to be a reasonable idea in order to get a smooth Social Distance curve in time. However, other filters may be considered, provided that doing so would lead to more precise results (to be analyzed).

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- ⁱ [https://en.wikipedia.org/wiki/Metric_\(mathematics\)#Quasimetrics](https://en.wikipedia.org/wiki/Metric_(mathematics)#Quasimetrics) data available on May 6th 2020 17:10
- ⁱⁱ https://en.wikipedia.org/wiki/Moving_average data available on May 6th 2020 18:30
- ⁱⁱⁱ <https://www.who.int/docs/default-source/coronaviruse/who-china-joint-mission-on-covid-19-final-report.pdf#:~:text=Using%20available%20preliminary%20data%2C,severe%20or%20critical%20disease>. Retrieved May 17th at 11:13 - Report of the WHO-China Joint Mission on Coronavirus Disease 2019 (COVID-19)
- ^{iv} https://www.nature.com/articles/s41586-020-2196-x_reference.pdf , Virological assessment of hospitalized patients with COVID-2019, Roman Wölfel et al., Retrieved May 18th at 11:16
- ^v https://en.wikipedia.org/wiki/COVID-19_pandemic_in_South_Korea data available on May 18th 2020 14:30
- ^{vi} <https://www.worldometers.info/coronavirus/> data available on May 18th 2020 14:30
- ^{vii} https://en.wikipedia.org/wiki/COVID-19_pandemic_in_Italy data available on May 18th 2020 15:30
- ^{viii} https://en.wikipedia.org/wiki/COVID-19_pandemic_in_the_Czech_Republic data available on May 18th 2020 16:00
- ^{ix} https://en.wikipedia.org/wiki/COVID-19_pandemic_in_Japan data available on May 18th 2020 16:40
- ^x https://en.wikipedia.org/wiki/COVID-19_pandemic_in_Romania retrieved May 19th 2020 at 10:30
- ^{xi} https://en.wikipedia.org/wiki/COVID-19_pandemic_in_Slovakia retrieved May 18th 2020 at 18:35
- ^{xii} https://en.wikipedia.org/wiki/COVID-19_pandemic_in_Spain retrieved May 19th 2020 at 9:30
- ^{xiii} https://en.wikipedia.org/wiki/COVID-19_pandemic_in_Hungary retrieved May 19th 2020 at 9:30
- ^{xiv} https://en.wikipedia.org/wiki/COVID-19_pandemic_in_France retrieved May 18th 2020 at 19:20
- ^{xv} https://en.wikipedia.org/wiki/COVID-19_pandemic_in_Germany retrieved May 18th 2020 at 18:45
- ^{xvi} https://en.wikipedia.org/wiki/COVID-19_pandemic_in_Switzerland retrieved May 19th 2020 at 9:45
- ^{xvii} https://en.wikipedia.org/wiki/COVID-19_pandemic_in_the_United_States retrieved May 2020 at 17:15